

Amendments to the Specification:

Please replace paragraph [0014] with the following amended paragraph:

[0014] The coherent detection process may be explained with several mathematic equations. The following description utilizes complex notation for sinusoids that are summarized in Appendix A. The electric field of the signal may be written as:

$$\text{Re}[E_s(t)e^{i\omega_s t}]$$

$$\underline{\text{Re}[E_s(t)e^{i\omega_s t}]}$$

Please replace paragraph [0015] with the following amended paragraph:

[0015] where ~~$E_s(t)$~~ $\underline{E_s(t)}$ is the slowly varying envelope containing the information encoded on amplitude and phase of the optical signal. Similarly, the electric field of the local oscillator may be described as:

Please replace paragraph [0016] with the following amended paragraph:

$$[0016] \quad \underline{\text{Re}[E_{LO}e^{i\omega_{LO}t}]}$$

$$\underline{\text{Re}[E_{LO}e^{i\omega_{LO}t}]}$$

Please replace paragraph [0017] with the following amended paragraph:

[0017] where ~~E_{LO}~~ $\underline{E_{LO}}$ is a constant for a local oscillator. The electric field of the light arriving at the photodetector 29 in the top branch of FIG. 1B (or the photodetector 24 in FIG. 1A) is the sum of the two electric fields:

$$E_1 = \text{Re}[E_s(t)e^{i\omega_s t} + E_{LO}e^{i\omega_{LO}t}]$$

$$\underline{E_1 = \text{Re}[E_s(t)e^{i\omega_s t} + E_{LO}e^{i\omega_{LO}t}]}$$

Please replace paragraph [0018] with the following amended paragraph:

[0018] and the optical power is:

$$\underline{P_1 = E_1^* E_1 = (E_s^*(t) e^{-i\omega_s t} + E_{LO}^* e^{-i\omega_{LO} t})(E_s(t) e^{i\omega_s t} + E_{LO} e^{i\omega_{LO} t})}$$

$$\underline{P_1 = |E_s(t)|^2 + |E_{LO}|^2 + 2 \operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}]} \quad (1)$$

$$\underline{P_1 = E_1^* E_1 = (E_s^*(t) e^{-i\omega_s t} + E_{LO}^* e^{-i\omega_{LO} t})(E_s(t) e^{i\omega_s t} + E_{LO} e^{i\omega_{LO} t})}$$

$$\underline{P_1 = |E_s(t)|^2 + |E_{LO}|^2 + 2 \operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}]} \quad (1)$$

Please replace paragraph [0019] with the following amended paragraph:

[0019] In the case of single ended detection, only one output of the combiner is used. $|E_{LO}|^2$ is constant with time. $|E_s(t)|^2$ is relatively small, given that the local oscillator power is much larger than the signal power. In addition, for the phase shift keying (PSK) and frequency shift keying (FSK) modulation formats $|E_s(t)|^2$ is constant with time. The dominant term in equation 1 is the beat term $\operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}]$.

Please replace paragraph [0020] with the following amended paragraph:

[0020] The output of the lower branch is the difference of the two electric fields, and the optical power is:

$$\underline{P_2 = |E_s(t)|^2 + |E_{LO}|^2 - 2 \operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}]} \quad (2)$$

$$\underline{P_2 = |E_s(t)|^2 + |E_{LO}|^2 - 2 \operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}]} \quad (2)$$

Please replace paragraph [0021] with the following amended paragraph:

[0021] The other mode of detection is balanced detection, where the electrical circuitry after the photodetectors evaluates the difference in photocurrent between the two detectors:

$$P_1 - P_2 = 4 \operatorname{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \right]$$

$$\underline{P_1 - P_2 = 4 \operatorname{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \right]}$$

Please replace paragraph [0024] with the following amended paragraph:

[0024] There are two modes of coherent detection: homodyne and heterodyne. With homodyne detection, the frequency difference between the signal and the local oscillator is zero. The local oscillator laser has to be phase locked to the incoming signal in order to achieve this. For homodyne detection the term $e^{i(\omega_s - \omega_{LO})t}$ $\underline{e^{i(\omega_s - \omega_{LO})t}}$ is 1, and the beat term becomes

$$\operatorname{Re} \left[E_s(t) E_{LO}^* \right]$$

$$\underline{\operatorname{Re} \left[E_s(t) E_{LO}^* \right]}$$

Please replace paragraph [0025] with the following amended paragraph:

[0025] For the binary phase shift keying (BPSK) modulation format, $E_s(t)$ $\underline{E_s(t)}$ takes on the value 1 or -1 depending on whether a logical "1" or "0" was transmitted, and the decision circuit can simply act on the beat term directly. Homodyne detection requires that the bandwidth of the photodetector and the subsequent components be close to the bit rate. In addition, homodyne detection gives a better sensitivity than any other way of detecting the signal. Also homodyne detection has an inherent ultranarrow optical filtering capability, in that all regions of the optical spectrum, which are more than the detector bandwidth away from the local oscillator, are rejected. This feature means that homodyne detection can support a higher density of WDM channels than by using passive optical filters for WDM demultiplexing. The homodyne detection method has the disadvantage that the local oscillator must be phase locked to the signal. The local oscillator and signal lasers must be narrow linewidth lasers, such as external cavity semiconductor lasers, which are typically more expensive than the distributed feedback (DFB) laser. Additionally, some polarization management methods do not work with homodyne detection.

Please replace paragraph [0026] with the following amended paragraph:

[0026] With heterodyne detection, there is a finite difference in optical frequency between the signal and local oscillator. All the amplitude and phase information on the signal appears on a carrier at angular frequency $(\omega_s - \omega_{LO})$ $(\omega_s - \omega_{LO})$, known as the intermediate frequency (IF), which can be detected using standard radio detection methods (e.g., synchronous detection, envelope detection or differential detection). Heterodyne detection has the advantage that the local oscillator does not need to be phase locked, and a DFB laser can be used for the LO and the signal lasers. Also it is possible to employ signal processing in the IF to compensate for chromatic dispersion, which is considered impossible to do with homodyne detection by existing techniques. The heterodyne detection processes suffer from the disadvantage that the difference frequency must be at least equal to half the optical spectral width of the signal, about 0.75 times the symbol rate, to avoid a penalty from self-imaging, which requires the bandwidth of the photodetector to be at least 1.5 times the symbol rate. The sensitivity of heterodyne detection is 3dB worse than homodyne detection. In addition, for heterodyning to work, there must be an empty region in the optical spectrum adjacent to the signal being detected, which constrains the density at which WDM channels can be packed.

Please replace paragraph [0050] with the following amended paragraph:

[0050] The present invention provides for recovery of information on an optical signal using a local oscillator which is not phase locked to the signal, and which may have an optical frequency arbitrarily close to that of the signal. FIG. 3A illustrates a simplified block diagram of a single ended quadrature sampling receiver 50 in the preferred embodiment of the present invention. A 90° hybrid passive unit 54 is depicted within the dotted box. The method of recovering a digital representation of the complex envelope of the signal electric field is known herein in the present invention as quadrature sampling. The 90° hybrid passive unit mixes the signal with a local oscillator 56 in two paths, such that the phase difference between the signal and LO 56 in one path differs from the phase difference in the other path by about 90°. There are many ways to make the 90° hybrid. As illustrated in FIG. 3A, the 90° hybrid passive unit splits

both the signal and LO and then combines in each output arm a replica of the signal with a replica of the LO. However, there is an extra path length in one arm of the LO splitter to apply the phase shift. The electric field of the local oscillator may be written as $\text{Re}[E_{LO} e^{i\omega_{LO} t}]$ $\text{Re}[E_{LO} e^{i\omega_{LO} t}]$ for the top path and $\text{Re}[iE_{LO} e^{i\omega_{LO} t}]$ $\text{Re}[iE_{LO} e^{i\omega_{LO} t}]$ for the bottom path, while the signal is $\text{Re}[E_s(t) e^{i\omega_s t}]$ $\text{Re}[E_s(t) e^{i\omega_s t}]$ in both paths. After mixing the LO with the signal, the beat term for the top path is, following equation 2 or 3:

$$\text{beat term 1} = \text{Re}[E_s(t) E_{LO} * e^{i(\omega_s - \omega_{LO})t}] \quad (4)$$

$$\text{beat term 1} = \text{Re}[E_s(t) E_{LO} * e^{i(\omega_s - \omega_{LO})t}] \quad (4)$$

Please replace paragraph [0051] with the following amended paragraph:

[0051] and for the lower arm:

$$\text{beat term 2} = \text{Re}[iE_s(t) E_{LO} * e^{i(\omega_s - \omega_{LO})t}]$$

$$\text{beat term 2} = \text{Im}[E_s(t) E_{LO} * e^{i(\omega_s - \omega_{LO})t}] \quad (5)$$

$$\text{beat term 2} = \text{Re}[iE_s(t) E_{LO} * e^{i(\omega_s - \omega_{LO})t}]$$

$$\text{beat term 2} = \text{Im}[E_s(t) E_{LO} * e^{i(\omega_s - \omega_{LO})t}] \quad (5)$$

Please replace paragraph [0052] with the following amended paragraph:

[0052] Two A/D converters 58 and 60, utilized after the photodetectors 62 and 64, in the two paths convert the photocurrents proportional to these two beat terms into a sequence of numerical values versus time. The digital signal processor unit 36 accepts inputs from the A/D converters in both paths. The DSP is capable of doing computations on complex numbers. The DSP is also able to calculate from its inputs, the complex envelope of the signal electric field, $E_s(t) E_s(t)$, using the following formula:

$$E_s(t) = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} [(beat\ term\ 1) + i(beat\ term\ 2)] \quad (6)$$

Please replace paragraph [0054] with the following amended paragraph:

[0054] ~~ϕ~~ ϕ is the argument (phase angle) of E_{LO} . This method of recovering a digital representation of a complex signal is known as quadrature sampling. This method may also be referred to as a heterodyne detection followed by synchronous demodulation using a complex local oscillator and digital phase estimation. Although quadrature sampling is used in radio communications, it has never been applied to the detection of an optical signal before. By combining sampled values from the two paths of the 90° hybrid passive unit into complex numbers, it is possible to perform heterodyne detection without problems from self-imaging even when the IF is much lower than the bit rate. Equation 6 assumes that the two beat terms are effectively sampled at the same instant. If the path lengths are not equal from the signal splitter to the two A/Ds, then this will cause timing skew. The DSP can compensate for the skew by using an elastic buffer store at one of its inputs.

Please replace paragraph [0055] with the following amended paragraph:

[0055] The rotating phasor in equation 6, $e^{-i(\omega_s + \omega_{LO})t} / E_{LO}^*$ $e^{-i(\omega_s - \omega_{LO})t} / E_{LO}^*$, contains $(\omega_s - \omega_{LO})t - \phi$, the phase of the signal with respect to the LO, which is not provided directly to the DSP and must be calculated by it from *beat term 1* and *beat term 2*. Only when the estimate of $(\omega_s - \omega_{LO})t - \phi$ is correct continuously over time (when the phase estimation algorithm is locked) may the data be recovered with a low bit error rate. After locking has occurred, the phase term wanders because of the finite linewidth of the signal and LO lasers over a time of typically many bit periods. The phase estimation algorithm must then track this phase wander. There are many types of phase estimation algorithm that can be implemented within the DSP, as described in "Digital Communications" by John G. Proakis (Proakis). If the modulation format of the incoming signal contains a pilot carrier then a digital phase

locked loop (PLL) or an open loop phase estimation algorithm can be applied to ~~(beat term 1) + i(beat term 2)~~ (beat term 1) + i(beat term 2) directly. An example of an open loop phase estimation algorithm is taking the arctangent of the ratio $(\text{beat term 2})/(\text{beat term 1})$ followed by a low pass filter function. When the signal's modulation format is such that it contains no carrier then ~~(beat term 1) + i(beat term 2)~~ (beat term 1) + i(beat term 2) must first be processed by a single spectral line generation function, such as a squaring function (or power law function in the case of high order PSK), or a decision directed multiplication, or in a Costas loop which combines the single spectral line generation function with the PLL. An alternative method of estimating the phase is to make use of known sequences of symbols that are repeated in the transmitted signal every time interval τ_{seq} , but this kind of method is useful only when the frequency difference between the signal and local oscillator is small, less than $1/2\tau_{seq}$.

Please replace paragraph [0058] with the following amended paragraph:

[0058] The values of $\text{Re}[E_s(t)]$ $E_s(t)$ and $\text{Im}[E_s(t)]$ $E_s(t)$ within the digital signal processor are the same as the detected optical powers that would be observed in the two arms of a conventional phase and quadrature homodyne detection system, such as with the QPSK receiver disclosed in "Linewidth requirements for optical synchronous detection systems with nonnegligible loop delay time," by S. Norimatsu and K. Iwashita (Norimatsu). Such a conventional receiver needs to have the local oscillator phase locked to the incoming optical signal, unlike the present invention. The digital information is obtained from $E_s(t)$ $E_s(t)$ by applying the function of a decision circuit within the DSP. For example, for BPSK E_s E_s takes on values [1,-1], and the decision circuit function operates on $\text{Re}[E_s]$ E_s with a threshold close to zero; $\text{Im}[E_s]$ E_s may then be ignored. For quadrature phase shift keying (QPSK), E_s E_s takes on values $[1+i, -1+i, 1-i, -1-i]$ $[1+i, -1+i, 1-i, -1-i]$, (i.e. two bits per symbol). Separate

decisions are made on $\underline{\text{Re}[E_s]}$ $\underline{\text{Re}[E_s]}$ and $\underline{\text{Im}[E_s]}$ $\underline{\text{Im}[E_s]}$ to give the two bits of content of the symbol.

Please replace paragraph [0059] with the following amended paragraph:

[0059] For some applications, such as the detection of a BPSK signal, the entire complex envelope of the electric field of the incoming signal does not need to be evaluated; only one component, such as the real part of the complex envelope, is wanted. Examples of a component of the complex envelope of the electric field are $\underline{\text{Re}[E_s(t)]}$ $\underline{\text{Re}[E_s(t)]}$, $\underline{\text{Im}[E_s(t)]}$ $\underline{\text{Im}[E_s(t)]}$ and $\underline{\text{Re}[E_s(t)e^{i\theta}]}$ $\underline{\text{Re}[E_s(t)e^{i\theta}]}$, where $\underline{\theta}$ is a constant.

Please replace paragraph [0061] with the following amended paragraph:

[0061] If the phase shift imposed by the hybrid is not 90° , but given by $e^{i\theta}$, the quadrature sampling process can still be applied by replacing equation 6 with

$$E_s(t) = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \left[(\text{beat term 1}) + i \left(\frac{(\text{beat term 2}) - \cos \theta (\text{beat term 1})}{\sin \theta} \right) \right] \quad (8)$$

Please replace paragraph [0062] with the following amended paragraph:

[0062] Only when $[\theta]$ is close to 0 or π (180°), does the quadrature detection scheme fail completely.

Please replace paragraph [0072] with the following amended paragraph:

[0072] The present invention may utilize a modification of this polarization diversity process. As discussed above, the signal and LO envelopes have been assigned complex scalar variables $\underline{E_s(t)}$ $\underline{E_s(t)}$ and $\underline{E_{LO}}$ $\underline{E_{LO}}$. The polarization nature is included by multiplying these scalar quantities by a Jones unit vector, so $\underline{E_s(t)}$ $\underline{E_s(t)}$ becomes $\underline{E_s(t)\hat{p}_s}$ $\underline{E_s(t)\hat{p}_s}$ and $\underline{E_{LO}}$ $\underline{E_{LO}}$ becomes $\underline{E_{LO}\hat{p}_{LO}}$ $\underline{E_{LO}\hat{p}_{LO}}$. The use of Jones vectors to represent polarization states is summarized in Appendix A. The result of coherent beating that appeared previously in equations 2 and 3 becomes:

$$\begin{aligned} \text{beat term} &= \text{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{LO}^* \right] \\ \text{beat term} &= \text{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{LO}^* \right] \end{aligned}$$

Please replace paragraph [0073] with the following amended paragraph:

[0073] $\hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{LO}^*$ $\hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{LO}^*$ is 1 when the LO and signal SOPs are aligned, and 0 when they are orthogonal. The LO has mutually orthogonal SOPs in the two polarization diversity paths, which may be represented as the (real) Jones unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$. The beat terms in the two paths are:

$$\text{beat term}_x = \text{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}} \right]$$

$$\text{beat term}_y = \text{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}} \right]$$

$$\text{beat term}_x = \text{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}} \right]$$

$$\text{beat term}_y = \text{Re} \left[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}} \right]$$

Please replace paragraph [0074] with the following amended paragraph:

[0074] The issue described above that prevents polarization diversity being used with homodyne detection may be summarized as follows: $\hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}$ $\hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}$ and $\hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}$ $\hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}$ are complex quantities whose phase can vary independently over time. With homodyne detection, the function of the phase locked loop on the local oscillator is to adjust the phase of E_{LO} E_{LO} to compensate for any phase changes in the incoming signal. However, it is not possible to keep constant both $E_{LO}^* \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}$ $E_{LO}^* \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}$ and $E_{LO}^* \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}$ $E_{LO}^* \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}$.

Please replace paragraph [0076] with the following amended paragraph:

[0076] In the preferred embodiment of the present invention, polarization tracking is achieved by adding polarization diversity to the phase diversity (90° hybrid

arrangement) discussed above. FIG. 5 is a simplified block diagram illustrating a polarization tracking system 131 in the preferred embodiment of the present invention. This configuration includes four photodetectors 110, 112, 114, and 116 (assuming single ended detection) and four A/D converters 120, 122, 124, 126, going to the DSP 130. If a LO 132 is divided into polarizations \hat{x} and \hat{y} , the four beat terms are:

$$\text{beat term } 1_x = \text{Re} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}] \quad (9a)$$

$$\text{beat term } 1_y = \text{Re} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}] \quad (9b)$$

$$\text{beat term } 2_x = \text{Im} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}] \quad (9c)$$

$$\text{beat term } 2_y = \text{Im} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}] \quad (9d)$$

$$\text{beat term } 1_x = \text{Re} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}] \quad (9a)$$

$$\text{beat term } 1_y = \text{Re} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}] \quad (9b)$$

$$\text{beat term } 2_x = \text{Im} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{x}}] \quad (9c)$$

$$\text{beat term } 2_y = \text{Im} [E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t} \hat{\mathbf{p}}_s \cdot \hat{\mathbf{y}}] \quad (9d)$$

Please replace paragraph [0077] with the following amended paragraph:

[0077] In the case discussed above where the polarization behavior was ignored, quadrature sampling was utilized by forming complex numbers from the two inputs to the DSP and processing them according to equation 6. With the polarization diversity configuration, a Jones vector is formed from the four inputs to the DSP, and the signal electric field is calculated from:

$$E_s(t) = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \left((beat\ term\ 1_x) + i(beat\ term\ 2_x) \right) \cdot \hat{\mathbf{p}}_s^* \quad (10)$$

$$E_s(t) = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \left((beat\ term\ 1_y) + i(beat\ term\ 2_y) \right) \cdot \hat{\mathbf{p}}_s^* \quad (10)$$

Please replace paragraph [0078] with the following amended paragraph:

[0078] To use equation 10 it is necessary to know $\hat{p}_s \underline{\hat{p}_s}$. This quantity can be estimated from:

$$\hat{p}_s = \sqrt{\frac{1}{1+|R|^2}} \begin{pmatrix} 1 \\ R \end{pmatrix}$$

$$\underline{\hat{p}_s} = \sqrt{\frac{1}{1+|R|^2}} \begin{pmatrix} 1 \\ R \end{pmatrix}$$

Please replace paragraph [0080] with the following amended paragraph:

[0080] The time average is preferably calculated over many bit periods to average out any additive amplified spontaneous emission noise. However, $\hat{p}_s \underline{\hat{p}_s}$ should be reevaluated sufficiently frequently so the changes in SOP of the incoming signal are tracked.

Please replace paragraph [0081] with the following amended paragraph:

[0081] With the present invention, this novel polarization diversity method is able to operate within the scenario that could not be tracked when using homodyne detection with existing methods. If $\hat{p}_s \cdot \hat{x} \underline{\hat{p}_s \cdot \hat{x}}$ and $\hat{p}_s \cdot \hat{y} \underline{\hat{p}_s \cdot \hat{y}}$ evolve in phase differently from one another, then equation 10 may still be applied. In such a circumstance, it will involve multiplying the top Jones vector element by a different phase factor from the bottom vector element. Because it is a mathematical multiplication that is done within the DSP 130, there is no constraint that the phase factor cannot be arbitrary as with an existing electro-optic phase modulator. Thus the present invention may be used with, for example, BPSK and QPSK modulation formats and provides the same sensitivity as for homodyne detection (i.e., the best possible sensitivity of any modulation format).

Please replace paragraph [0082] with the following amended paragraph:

[0082] The application of the polarization diverse process of FIG. 4A provides more than just a complex representation of the signal that is calculated within the DSP

130, but additionally, it is a Jones vector representation that is obtained, which contains all the polarization information. The Jones vector $\underline{\underline{E}_s(t)}$ $\underline{\underline{E}_s(t)}$ is given by:

$$\underline{\underline{E}_s(t)} = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \begin{pmatrix} ((\text{beat term } 1_x) + i(\text{beat term } 2_x)) \\ ((\text{beat term } 1_y) + i(\text{beat term } 2_y)) \end{pmatrix} \quad (11)$$

$$\underline{\underline{E}_s(t)} = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \begin{pmatrix} ((\text{beat term } 1_x) + i(\text{beat term } 2_x)) \\ ((\text{beat term } 1_y) + i(\text{beat term } 2_y)) \end{pmatrix} \quad (11)$$

Please replace paragraph [0084] with the following amended paragraph:

[0084] The passive unit 54 discussed above combines the signal and local oscillator into four different arms. The SOPs of the local oscillator in two arms are orthogonal relative to the other two arms (polarizations \hat{x} \hat{x} and \hat{y} \hat{y}). Thus, a pair of arms having the same SOP also have phases that are 90° apart. The Jones vectors of the LO in the four arms are:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} i \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ i \end{pmatrix} \quad (12)$$

Please replace paragraph [0088] with the following amended paragraph:

[0088] Following the notation that the x-component of a Jones vector is denoted by adding suffix x, etc., with the Jones vectors of the LO in the four arms is \hat{p}_1 \hat{p}_1 , \hat{p}_2 \hat{p}_2 , \hat{p}_3 \hat{p}_3 and \hat{p}_4 \hat{p}_4 , and the corresponding photodetector outputs are beat term 1 .. beat term 4, then the signal can be calculated from:

$$\begin{pmatrix} \text{Re}[E_{sx}(t)] \\ \text{Im}[E_{sx}(t)] \\ \text{Re}[E_{sy}(t)] \\ \text{Im}[E_{sy}(t)] \end{pmatrix} = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \begin{pmatrix} \text{Re}[\hat{p}_{1x}] & \text{Im}[\hat{p}_{1x}] & \text{Re}[\hat{p}_{1y}] & \text{Im}[\hat{p}_{1y}] \\ \text{Re}[\hat{p}_{2x}] & \text{Im}[\hat{p}_{2x}] & \text{Re}[\hat{p}_{2y}] & \text{Im}[\hat{p}_{2y}] \\ \text{Re}[\hat{p}_{3x}] & \text{Im}[\hat{p}_{3x}] & \text{Re}[\hat{p}_{3y}] & \text{Im}[\hat{p}_{3y}] \\ \text{Re}[\hat{p}_{4x}] & \text{Im}[\hat{p}_{4x}] & \text{Re}[\hat{p}_{4y}] & \text{Im}[\hat{p}_{4y}] \end{pmatrix}^{-1} \begin{pmatrix} \text{beat term 1} \\ \text{beat term 2} \\ \text{beat term 3} \\ \text{beat term 4} \end{pmatrix} \quad (13)$$

$$\begin{pmatrix} \text{Re}[E_{sx}(t)] \\ \text{Im}[E_{sx}(t)] \\ \text{Re}[E_{sy}(t)] \\ \text{Im}[E_{sy}(t)] \end{pmatrix} = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \begin{pmatrix} \text{Re}[\hat{p}_{1x}] & \text{Im}[\hat{p}_{1x}] & \text{Re}[\hat{p}_{1y}] & \text{Im}[\hat{p}_{1y}] \\ \text{Re}[\hat{p}_{2x}] & \text{Im}[\hat{p}_{2x}] & \text{Re}[\hat{p}_{2y}] & \text{Im}[\hat{p}_{2y}] \\ \text{Re}[\hat{p}_{3x}] & \text{Im}[\hat{p}_{3x}] & \text{Re}[\hat{p}_{3y}] & \text{Im}[\hat{p}_{3y}] \\ \text{Re}[\hat{p}_{4x}] & \text{Im}[\hat{p}_{4x}] & \text{Re}[\hat{p}_{4y}] & \text{Im}[\hat{p}_{4y}] \end{pmatrix}^{-1} \begin{pmatrix} \text{beat term 1} \\ \text{beat term 2} \\ \text{beat term 3} \\ \text{beat term 4} \end{pmatrix} \quad (13)$$

Please replace paragraph [0089] with the following amended paragraph:

[0089] Equation 11 is a special case of equation 13 for \hat{p}_1 to \hat{p}_4 given by equation 12. The four \hat{p}_i Jones vectors must be distinct from one another. If one of the \hat{p}_i is equal to another, or -1 multiplied by another (180° phase shift), then the 4x4 matrix in equation 13 cannot be inverted. Therefore, $E_s(t)$ cannot be determined.

Please replace paragraph [0090] with the following amended paragraph:

[0090] Thus, the quadrature sampling process may be used to determine the amplitude, phase, and polarization information of a signal. An apparatus is used which contains a local oscillator and four independent photodetectors. Each photodetector is exposed to a sum of signal light and local oscillator light. The optical phase of the LO compared to the signal or the state of polarization of the LO compared to the signal must be distinct between the inputs to the four photodetectors. Specifically, no two photodetectors must see substantially the same relative phase and the same relative SOP. (A phase difference of 180° is considered to be the same as 0° . The requirement is that the four photodetectors see distinct Jones vectors of the LO compared to the signal.) Two Jones vectors \hat{p}_1 and \hat{p}_2 are defined as being not distinct if there is a real number K such that $\hat{p}_1 = K\hat{p}_2$ such that $\hat{p}_1 = K\hat{p}_2$. An apparatus can be used that employs more than four photodetectors, if it is possible to select four photodetectors from the total that have distinct Jones vectors of the LO relative to the signal.

Please replace paragraph [0093] with the following amended paragraph:

[0093] Another way that the quadrature sampling process may be used to obtain a representation of the signal, including its polarization information, employs two different local oscillators having different optical frequencies (see FIG. 4B). One of the LOs is split into two paths with different phases, but with the same SOP. The other LO is again split into two paths having different phases and the same SOP, but the SOP of the second pair of paths is close to orthogonal to the SOP of the first pair of paths. The Jones vector of the signal can be obtained by using a version of equation 11. If the first pair of paths has SOP \hat{x} and LO optical frequency ω_{LOx} , and the second pair of paths has \hat{y} and ω_{LOy} , then

$$\mathbf{E}_s(t) = \frac{1}{E_{LO}} * \left(\frac{e^{-i(\omega_s - \omega_{LOx})t} ((\text{beat term } 1_x) + i(\text{beat term } 2_x))}{e^{-i(\omega_s - \omega_{LOy})t} ((\text{beat term } 1_y) + i(\text{beat term } 2_y))} \right)$$

$$\mathbf{E}_s(t) = \frac{1}{E_{LO}} * \left(\frac{e^{-i(\omega_s - \omega_{LOx})t} ((\text{beat term } 1_x) + i(\text{beat term } 2_x))}{e^{-i(\omega_s - \omega_{LOy})t} ((\text{beat term } 1_y) + i(\text{beat term } 2_y))} \right)$$

Please replace paragraph [0095] with the following amended paragraph:

[0095] It is possible to use the polarization management features described here without using quadrature sampling. For example homodyne detection can be used with a polarization and phase diversity configuration whose outputs are digitized and processed by a DSP. In this case equations 10, 11 or 13 can be applied with $\omega_s - \omega_{LO} = 0$. Alternatively, conventional heterodyne detection can be used in conjunction with polarization diversity. The use of the DSP provides the same advantage described previously. Specifically, the incoming SOP can effectively be tracked endlessly even though homodyne detection is used, because the DSP is able to apply an arbitrary phase shift.

Please replace paragraph [0097] with the following amended paragraph:

[0097] The polarization tracking system 131 of FIG. 5 may be utilized to demultiplex two polarization multiplexed signals. Jones vector manipulation within the

DSP may be used to emulate the effect of inserting a polarizer in the optical signal path. In general, if the input to a polarizer is $E_s \hat{\mathbf{p}}_s$ and the polarizer has maximum transmission state $\hat{\mathbf{p}}_{pol} \hat{\mathbf{p}}_{pol}^*$, then the electric field at the output of the polarizer is $E_s (\hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{pol}^* \hat{\mathbf{p}}_{pol}) E_s (\hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{pol}^*) \hat{\mathbf{p}}_{pol}$, assuming no excess loss. The value of the electric field passing through the polarizer may be derived by using the following variant of equation 10:

$$E_s(t) \hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{pol}^* = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \left(\begin{array}{l} (\text{beat term } 1_x) + i(\text{beat term } 2_x) \\ (\text{beat term } 1_y) + i(\text{beat term } 2_y) \end{array} \right) \cdot \hat{\mathbf{p}}_{pol}^* \quad (14)$$

$$E_s(t) \hat{\mathbf{p}}_s \cdot \hat{\mathbf{p}}_{pol}^* = \frac{e^{-i(\omega_s - \omega_{LO})t}}{E_{LO}^*} \left(\begin{array}{l} (\text{beat term } 1_x) + i(\text{beat term } 2_x) \\ (\text{beat term } 1_y) + i(\text{beat term } 2_y) \end{array} \right) \cdot \hat{\mathbf{p}}_{pol}^* \quad (14)$$

Please replace paragraph [0099] with the following amended paragraph:

[0099] The electric field of the multiplexed channels, A and B, may be written as:

$$\begin{aligned} & \text{Re} \left[E_{sA}(t) e^{i\omega_{sA} t} \hat{\mathbf{p}}_{sA} + E_{sB}(t) e^{i\omega_{sB} t} \hat{\mathbf{p}}_{sB} \right] \\ & \text{Re} \left[E_{sA}(t) e^{i\omega_{sA} t} \hat{\mathbf{p}}_{sA} + E_{sB}(t) e^{i\omega_{sB} t} \hat{\mathbf{p}}_{sB} \right] \end{aligned}$$

Please replace paragraph [0100] with the following amended paragraph:

[0100] $\hat{\mathbf{p}}_{sA} \hat{\mathbf{p}}_{sA}^*$ and $\hat{\mathbf{p}}_{sB} \hat{\mathbf{p}}_{sB}^*$ are the Jones unit vectors of the SOPs of A and B.

To recover channel A, the signal must be effectively passed through a polarizer oriented to be orthogonal to channel B, that is $\hat{\mathbf{p}}_{sB}^\dagger \hat{\mathbf{p}}_{sB}^\dagger$. Channel A is recovered by applying equation 14:

$$E_{sA}(t) = \frac{e^{-i(\omega_{sA} - \omega_{LO})t}}{E_{LO}^* (\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sB}^*)^*} \left(\begin{array}{l} (\text{beat term } 1_x) + i(\text{beat term } 2_x) \\ (\text{beat term } 1_y) + i(\text{beat term } 2_y) \end{array} \right) \cdot \hat{\mathbf{p}}_{sB}^* \quad (15)$$

$$E_{sA}(t) = \frac{e^{-i(\omega_{sA} - \omega_{LO})t}}{E_{LO}^* (\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sB}^*)^*} \left(\begin{array}{l} (\text{beat term } 1_x) + i(\text{beat term } 2_x) \\ (\text{beat term } 1_y) + i(\text{beat term } 2_y) \end{array} \right) \cdot \hat{\mathbf{p}}_{sB}^* \quad (15)$$

Please replace paragraph [0101] with the following amended paragraph:

[0101] ~~$\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sB}^{\dagger*}$~~ ~~$\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sB}^{\dagger*}$~~ is nearly 1, given that A and B are close to orthogonal, and can, therefore, be ignored. In a similar manner, to recover channel B, a polarizer ~~$\hat{\mathbf{p}}_{sA}^{\dagger}$~~ ~~$\hat{\mathbf{p}}_{sA}^{\dagger}$~~ is utilized:

$$E_{sB}(t) = \frac{e^{-i(\omega_{sB}-\omega_{LO})t}}{E_{LO} * (\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sA}^{\dagger*})} \left(\begin{array}{l} (\text{beat term } 1_x) + i(\text{beat term } 2_x) \\ (\text{beat term } 1_y) + i(\text{beat term } 2_y) \end{array} \right) \cdot \hat{\mathbf{p}}_{sA}^{\dagger*} \quad (16)$$

$$E_{sB}(t) = \frac{e^{-i(\omega_{sB}-\omega_{LO})t}}{E_{LO} * (\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sA}^{\dagger*})} \left(\begin{array}{l} (\text{beat term } 1_x) + i(\text{beat term } 2_x) \\ (\text{beat term } 1_y) + i(\text{beat term } 2_y) \end{array} \right) \cdot \hat{\mathbf{p}}_{sA}^{\dagger*} \quad (16)$$

Please replace paragraph [0102] with the following amended paragraph:

[0102] The polarization multiplexed channels may be separated even if they are not perfectly orthogonal. They can be separated without the use of any extra hardware. The same polarization diversity tracking system 131 may be employed. To use equations 15 and 16, ~~$\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sA}^{\dagger}$~~ and ~~$\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sB}^{\dagger}$~~ must be known. ~~$\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sA}^{\dagger}$~~ and ~~$\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sB}^{\dagger}$~~ may be determined by an adaptive process which explores all of the polarization space. When the value of ~~$\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sB}^{\dagger}$~~ is close to the correct value, it is possible to recover channel A recognizably. The bit error rate of A may then be used as a metric to obtain the exact value of ~~$\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sB}^{\dagger}$~~ . When ~~$\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sA}^{\dagger}$~~ and ~~$\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sB}^{\dagger}$~~ are known correctly, then each signal may be recovered without crosstalk from the other signal. ~~$\hat{\mathbf{p}}_{sA} \cdot \hat{\mathbf{p}}_{sA}^{\dagger}$~~ and ~~$\hat{\mathbf{p}}_{sB} \cdot \hat{\mathbf{p}}_{sB}^{\dagger}$~~ must be allowed to track the slow variation in incoming SOPs due to the environmental disturbances experienced by the fiber link. The process of setting the orientation of a real (physical) polarization demultiplexer must also be conducted by trial and error and is relatively slow. The present invention provides the advantage that the iteration speed is determined by the computation time within the DSP, and not by the reaction time of any polarization control hardware.

Please replace paragraph [0110] with the following amended paragraph:

[0110] The quadrature sampling process provides a complete representation of the optical signal (i.e., amplitude, phase, and state of polarization). All other parameters may be derived from this basic information. With this complete representation of the signal, the result can be calculated for any deterministic physical process that happens in the transmission optical fiber or the terminal electronics, provided the calculation is within the computation ability of the DSP. Any deterministic impairment can be reversed by a calculation within the DSP. The term "deterministic process" does not include the addition of noise, or the subtraction of added noise. It is not possible to fully reverse all impairments using an equalizer (DSP or ASP) after direct detection, although such an equalizer can improve the signal. The result of the direct detection operation is $\underline{|E_s(t)|^2} \underline{|E_s(i)|^2}$, and the phase and polarization information has been discarded.

Please replace paragraph [0115] with the following amended paragraph:

[0115] For chromatic dispersion (CD) on an optical signal, the CD of a section of fiber is described by the 2nd order group delay coefficient β_2 and the fiber length L . This is disclosed in "Nonlinear fiber optics" by G. Agrawal (Agrawal 1). If the electric field envelope at the input to the fiber is $E_{in}(t)$, then the Fourier transform is denoted by $\tilde{E}_{in}(\omega)$, and similarly for the output field $E_{out}(t)$. Ignoring the effect of fiber loss, the impact of chromatic dispersion alone is:

$$\tilde{E}_{out}(\omega) = \tilde{E}_{in}(\omega) e^{i\frac{1}{2}\beta_2\omega^2 L}$$

$$\underline{\tilde{E}_{out}(\omega) = \tilde{E}_{in}(\omega) e^{i\frac{1}{2}\beta_2\omega^2 L}}$$

Please replace paragraph [0116] with the following amended paragraph:

[0116] Inverting this relationship results in:

$$\tilde{E}_{in}(\omega) = \tilde{E}_{out}(\omega) e^{-i\frac{1}{2}\beta_2\omega^2 L} \quad (17)$$

$$\underline{\tilde{E}_{in}(\omega) = \tilde{E}_{out}(\omega) e^{-i\frac{1}{2}\beta_2\omega^2 L}} \quad (17)$$

Please replace paragraph [0117] with the following amended paragraph:

[0117] Equation 17 deals with the Fourier transform of the signal, i.e. it expresses a linear filter relationship. Denoting the filter function by $\tilde{f}(\omega)$ $\tilde{f}(\omega)$

$$\tilde{f}(\omega) = e^{-i\frac{1}{2}\beta_2\omega^2 L}$$

$$\tilde{f}(\omega) = e^{-i\frac{1}{2}\beta_2\omega^2 L}$$

Please replace paragraph [0118] with the following amended paragraph:

[0118] and its inverse Fourier transform $f(t)$ $f(t)$ can be calculated. Then:

$$E_{in}(t) = E_{out}(t) \otimes f(t) \quad (18)$$

$$E_{in}(t) = E_{out}(t) \otimes f(t) \quad (18)$$

Please replace paragraph [0119] with the following amended paragraph:

[0119] (~~\otimes denotes the convolution operation.~~) (\otimes denotes the convolution operation.) Equation 18 may be applied by the DSP, and, in principle, compensates perfectly for the chromatic dispersion of the fiber section. The DSP can convolve only a finite length vector $f(t)$ $f(t)$, and so it has to be truncated according to the computation ability of the DSP. $f(t)$ $f(t)$ takes the form of a resonance with high magnitude points close to $t=0$ $t=0$, so the truncation should not lead to a large error. If $\beta_2 L$ $\beta_2 L$ of the link is not known, it can be found adaptively, such as by updating the vector $f(t)$ $f(t)$ by trial and error to obtain the best result.

Please replace paragraph [0122] with the following amended paragraph:

[0122] Oftentimes, most of the system penalty comes from first order PMD. If the input to a section of fiber is $E_{in}(t)\hat{p}_{in}$ $E_{in}(t)\hat{p}_{in}$ (constant in SOP with time), the output is $E_{out}(t)$ $E_{out}(t)$ (not necessarily having constant SOP with time). In addition, the

principal states have Jones vectors $\hat{\mathbf{p}}_{PMD}$ $\hat{\mathbf{p}}_{PMD}^*$ and $\hat{\mathbf{p}}_{PMD}^\dagger$ $\hat{\mathbf{p}}_{PMD}^{\dagger\dagger}$, and the first order PMD is τ , then ignoring the SOP transformation of the fiber section and ignoring the fiber loss, the impact of the first order PMD is:

$$\mathbf{E}_{out}(t) = E_{in}(t)(\hat{\mathbf{p}}_{in} \cdot \hat{\mathbf{p}}_{PMD}^*) \hat{\mathbf{p}}_{PMD} + E_{in}(t-\tau)(\hat{\mathbf{p}}_{in} \cdot \hat{\mathbf{p}}_{PMD}^{\dagger\dagger}) \hat{\mathbf{p}}_{PMD}^\dagger \quad (19)$$

Please replace paragraph [0123] with the following amended paragraph:

[0123] Equation 19 indicates that the signal is separated in two according to how much of the signal lies in the two principal states and one state is retarded in time by τ while the other is left alone. Equation 19 is reversed by:

$$E_{in}(t) = E_{out}(t) \cdot \hat{\mathbf{p}}_{PMD}^* (\hat{\mathbf{p}}_{in} \cdot \hat{\mathbf{p}}_{PMD}^*) + E_{out}(t+\tau) \cdot \hat{\mathbf{p}}_{PMD}^{\dagger\dagger} (\hat{\mathbf{p}}_{in} \cdot \hat{\mathbf{p}}_{PMD}^{\dagger\dagger}) \quad (20)$$

$$E_{in}(t) = (E_{out}(t) \cdot \hat{\mathbf{p}}_{PMD}^*) (\hat{\mathbf{p}}_{in}^* \cdot \hat{\mathbf{p}}_{PMD}) + (E_{out}(t+\tau) \cdot \hat{\mathbf{p}}_{PMD}^{\dagger\dagger}) (\hat{\mathbf{p}}_{in}^* \cdot \hat{\mathbf{p}}_{PMD}^\dagger) \quad (20)$$

Please replace paragraph [0124] with the following amended paragraph:

[0124] This relationship may be implemented by the DSP so as to compensate for the first order PMD. τ is typically not a whole multiple of the sampling interval of the A/D. Therefore, it is necessary to perform interpolation to obtain both $E_{out}(t)$ $E_{out}(t)$ and $E_{out}(t+\tau)$. $\hat{\mathbf{p}}_{PMD}$ $\hat{\mathbf{p}}_{PMD}^*$ and τ are not known initially to the DSP and must be found adaptively. These values vary with time and, therefore, the DSP must track the real values.

Please replace paragraph [0127] with the following amended paragraph:

[0127] Multipath interference occurs when an optical signal is split into two or more paths having different physical lengths and then recombined. Usually one path (i.e., the main path) carries a much stronger signal than the others, but the power in the "echoes" arriving via the other path degrades the signal that has traveled through the main path. FIG. 7A is a simplified block diagram illustrating a subsystem 300 that contributes MPI, containing a pair of optical amplifiers 302 and 304 connected in parallel. The two optical amplifiers have different passbands. FIG. 7B is a graphical representation of an associated gain spectrum for FIG. 7A. The configuration in FIG.

7A is used to produce amplification over an extended optical bandwidth. However, it is not possible to operate over a continuous bandwidth because there is an unusable region in between the two passbands where the MPI is too great. At a wavelength in the dead zone, typically one amplifier has more gain, called G_1 , but the gain of the other amplifier, G_2 , is sufficiently large that MPI is a problem. The impact of this situation of two path MPI on the signal electric field is (ignoring the effect of the optical polarization):

$$E_{out}(t) = \sqrt{G_1} E_{in}(t) + \sqrt{G_2} e^{-i\omega_s \tau_{MPI}} E_{in}(t - \tau_{MPI}) \quad (21)$$

$$\underline{E_{out}(t) = \sqrt{G_1} E_{in}(t) + \sqrt{G_2} e^{-i\omega_s \tau_{MPI}} E_{in}(t - \tau_{MPI})} \quad (21)$$

Please replace paragraph [0128] with the following amended paragraph:

[0128] where τ_{MPT} τ_{MPI} is the difference in time delay of the two paths.

Please replace paragraph [0130] with the following amended paragraph:

[0130] where τ_{samp} τ_{samp} is the sampling interval of the A/D converter and it is assumed that τ_{MPT} τ_{MPI} is a whole multiple of τ_{samp} τ_{samp} . The MPI impairment is reversed by:

$$E_{in}(z) = \frac{1}{\sqrt{G_1} + \sqrt{G_2} e^{-i\omega_s \tau_{MPI}} z^{-\frac{\tau_{MPI}}{\tau_{samp}}}} E_{out}(z) \quad (22)$$

Please replace paragraph [0131] with the following amended paragraph:

[0131] MPI compensation can be achieved by implementing a digital filter within the DSP. As explained in Lyons, the output $y(n)$ of a general recursive digital filter is computed from the input $x(n)$ by:

$$y(n) = \sum_{k=0}^{\infty} a_k x(n-k) + \sum_{k=0}^{\infty} b_k y(n-k)$$

$$y(n) = \sum_{k=0}^{\infty} a_k x(n-k) + \sum_{k=1}^{\infty} b_k y(n-k)$$

Please replace paragraph [0132] with the following amended paragraph:

[0132] The transfer function of equation 22 may be obtained from a digital filter using these coefficients:

$$a_0 = \frac{1}{\sqrt{G_1}}$$

$$\underline{a_k = 0} \quad k > 0$$

$$\underline{a_k = 0} \quad k > 0$$

Please replace paragraph [0133] with the following amended paragraph:

[0133] $b_k = -\sqrt{\frac{G_2}{G_1}} e^{-i\omega_s \tau_{MPI}}$ $k = \frac{\tau_{MPI}}{\tau_{samp}}$

$$\underline{b_k = 0} \quad \text{otherwise}$$

$$\underline{b_k = 0} \quad \text{otherwise}$$

Please replace paragraph [0134] with the following amended paragraph:

[0134] τ_{MPI} and G_2/G_1 can be found adaptively by the DSP given that they are not known initially.

Please replace paragraph [0139] with the following amended paragraph:

[0139] When coherent detection is used, it is equivalent to inserting a narrow optical filter centered around the local oscillator wavelength. The passband shape of the filter is symmetric, and is given by unfolding around $\omega=0$ the baseband response of the detector + amplifier chain + other components prior to the decision circuit (in the present invention, the A/D converter + DSP). Within the preferred

embodiment of the present invention, the effective filter shape may be tailored within the DSP. An adaptive filter may be incorporated in the DSP first to undo any unwanted features in the frequency response of the detector etc.

Please replace paragraph [0141] with the following amended paragraph:

[0141] For example, assume that the desired filter shape is $\tilde{f}(\omega - \omega_{LO})$. The filter shape does not have to be symmetric about $\omega - \omega_{LO} = 0$, as it would if an analog signal processing stage were used. The constraint does not apply because the DSP is processing complex electric field values. Assuming that the detector + amp chain effectively has a flat frequency response, the optical filter is implemented by:

$$\underline{E_s}_{filt}(t) = E_s(t) \otimes f(t)$$

$$\underline{E_s}_{filt}(t) = E_s(t) \otimes f(t)$$

Please replace paragraph [0142] with the following amended paragraph:

[0142] where $f(t)$ is the inverse Fourier transform of $\tilde{f}(\omega)$.

Please replace paragraph [0144] with the following amended paragraph:

[0144] Recalling that the optical power in one arm of a coherent detection apparatus is given by equation 1:

$$P_1 = |E_s(t)|^2 + |E_{LO}|^2 + 2 \operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}] \quad (1)$$

$$\underline{P_1} = |E_s(t)|^2 + |E_{LO}|^2 + 2 \operatorname{Re}[E_s(t) E_{LO}^* e^{i(\omega_s - \omega_{LO})t}] \quad (1)$$

Please replace paragraph [0145] with the following amended paragraph:

[0145] The local oscillator power $|E_{LO}|^2$ is constant, so the detected power is equal to the beat term only if the local oscillator power is considerably larger than the signal power. Differential detection allows the $|E_s(t)|^2$ term to be subtracted. However, it is preferred to utilize single ended detection because it saves the cost of a second photodetector.

Please replace paragraph [0146] with the following amended paragraph:

[0146] It is possible to deduce $E_s(t)$ from the two phase diverse arms of a quadrature sampling receiver, even if the LO power is not much larger than the signal power. A first estimate of the signal envelope $E_{s\ est1}(t)$ is made using the method for quadrature sampling described above. This first estimate will be related to the true value $E_s(t)$ as follows

$$E_{s\ est1}(t) = E_s(t) + |E_s(t)|^2 \frac{1+i}{2|E_{LO}|} e^{-i(\omega_s - \omega_{LO})t} \quad (23)$$

Please replace paragraph [0147] with the following amended paragraph:

[0147] The proportional deviation of $E_{s\ est1}(t)$ from $E_s(t)$ (the second term in equation 23) has magnitude

$$\frac{|E_s(t)|}{\sqrt{2}|E_{LO}|}.$$

Please replace paragraph [0149] with the following amended paragraph:

[0149] $E_{s\ est2}$ is closer to E_s than $E_{s\ est1}$ provided that the power of the LO is greater than that of the signal. In actuality, $E_{s\ est2}$ is related to E_s by:

$$\begin{aligned} E_{s \text{ est2}}(t) &= E_s(t) - \frac{E_s(t)|E_s(t)|^2}{2|E_{LO}|^2} - \frac{iE_s^*(t)|E_s(t)|^2}{2|E_{LO}|^2} e^{-i2(\omega_s-\omega_{LO})t} & (1+i)|E_s(t)|^4 e^{-i(\omega_s-\omega_{LO})t} \\ E_{s \text{ est2}}(t) &= E_s(t) - \frac{E_s(t)|E_s(t)|^2}{2|E_{LO}|^2} - \frac{iE_s^*(t)|E_s(t)|^2}{2|E_{LO}|^2} e^{-i2(\omega_s-\omega_{LO})t} & 4|E_{LO}|^* |E_{LO}|^2 \\ & & (1+i)|E_s(t)|^4 e^{-i(\omega_s-\omega_{LO})t} \end{aligned}$$

Please replace paragraph [0150] with the following amended paragraph:

[0150] The proportional deviation of $\underline{E_{s \text{ est2}}(t)}$ $\underline{E_{s \text{ est2}}(t)}$ from $\underline{E_s(t)}$ $\underline{E_s(t)}$ now has magnitude

$$\frac{|E_s(t)|^2}{|E_{LO}|^2},$$

Please replace paragraph [0151] with the following amended paragraph:

[0151] which is smaller than it was for $\underline{E_{s \text{ est1}}}$ $\underline{E_{s \text{ est1}}}$. Repeated iterations of equation 24 yield an estimate of $\underline{E_s}$ $\underline{E_s}$ that is closer to the actual value.

Please replace paragraph [0158] with the following amended paragraph:

[0158] To detect two channels, centered on $\underline{\omega_{sA}}$ $\underline{\omega_{sA}}$ and $\underline{\omega_{sB}}$ $\underline{\omega_{sB}}$, using the same receiver, equation 6 is used with different signal center frequencies and a filter function $\tilde{f}(\omega)$ $\underline{\tilde{f}(\omega)}$ (having inverse Fourier transform $f(t)$) $\underline{f(t)}$ is applied to remove the other channel as shown below:

$$E_{sA}(t) = \left(\frac{e^{-i(\omega_{sA}-\omega_{LO})t}}{|E_{LO}|^*} [(beat \ term \ 1) + i(beat \ term \ 2)] \right) \otimes f(t) \quad (25a)$$

$$E_{sB}(t) = \left(\frac{e^{-i(\omega_{sB}-\omega_{LO})t}}{|E_{LO}|^*} [(beat \ term \ 1) + i(beat \ term \ 2)] \right) \otimes f(t) \quad (25b)$$

$$E_{sA}(t) = \left(\frac{e^{-i(\omega_{sA} - \omega_{LO})t}}{E_{LO}^*} [(beat\ term\ 1) + i(beat\ term\ 2)] \right) \otimes f(t) \quad (25a)$$

$$E_{sB}(t) = \left(\frac{e^{-i(\omega_{sB} - \omega_{LO})t}}{E_{LO}^*} [(beat\ term\ 1) + i(beat\ term\ 2)] \right) \otimes f(t) \quad (25b)$$

Please replace paragraph [0161] with the following amended paragraph:

[0161] For two WDM channels, A and B , in the presence of additive noise from optical amplifiers $n(t)$, the signal electric field (ignoring SOP) arriving at the receiver can be written as:

$$\begin{aligned} & \text{Re}[E_A(t)e^{i\omega_At} + E_B(t)e^{i\omega_Bt} + n(t)] \\ & \underline{\text{Re}[E_A(t)e^{i\omega_At} + E_B(t)e^{i\omega_Bt} + n(t)]} \end{aligned}$$

Please replace paragraph [0162] with the following amended paragraph:

[0162] A first estimate of each of the channels may be made using quadrature sampling, either in one receiver or in two separate receivers with different LOs, and applying a narrow filter having impulse response $f(t)$, per equations 25. The estimates will each contain crosstalk from the other channel.

Please replace paragraph [0163] with the following amended paragraph:

$$E_{A\ est1}(t) = E_A(t) \otimes f(t) + (n(t)e^{-i\omega_At}) \otimes f(t) + (E_B(t)e^{i(\omega_B - \omega_A)t}) \otimes f(t) \quad (26a)$$

$$E_{B\ est1}(t) = E_B(t) \otimes f(t) + (n(t)e^{-i\omega_Bt}) \otimes f(t) + (E_A(t)e^{i(\omega_A - \omega_B)t}) \otimes f(t) \quad (26b)$$

$$E_{A\ est1}(t) = E_A(t) \otimes f(t) + (n(t)e^{-i\omega_At}) \otimes f(t) + (E_B(t)e^{i(\omega_B - \omega_A)t}) \otimes f(t) \quad (26a)$$

$$E_{B\ est1}(t) = E_B(t) \otimes f(t) + (n(t)e^{-i\omega_Bt}) \otimes f(t) + (E_A(t)e^{i(\omega_A - \omega_B)t}) \otimes f(t) \quad (26b)$$

Please replace paragraph [0164] with the following amended paragraph:

[0164] The first term of the right hand side of equations 26a and 26b is the filtered version of the desired signal, the second term is noise, and the third term is crosstalk. The presence of crosstalk means that the bit error rate will be higher than it would be if the other channel were not there. The two estimates can be passed to a decision function, denoted by $\underline{Q}(\cdot)$, which predicts which of the allowed values was transmitted. The quantized value of each channel, $\underline{Q}(E_{est\ A})$ or $\underline{Q}(E_{est\ B})$, can then be subtracted from the other channel to make a second estimate:

$$E_{A\ est2}(t) = E_{A\ est1}(t) - \left(\underline{Q}(E_{B\ est1}(t)) e^{i(\omega_B - \omega_A)t} \right) \otimes f(t) \quad (27a)$$

~~$$E_{B\ est2}(t) = E_{B\ est1}(t) - \left(\underline{Q}(E_{A\ est1}(t)) e^{i(\omega_A - \omega_B)t} \right) \otimes f(t) \quad (27b)$$~~

~~$$E_{A\ est2}(t) = E_{A\ est1}(t) - \left(\underline{Q}(E_{B\ est1}(t)) e^{i(\omega_B - \omega_A)t} \right) \otimes f(t) \quad (27a)$$~~

~~$$E_{B\ est2}(t) = E_{B\ est1}(t) - \left(\underline{Q}(E_{A\ est1}(t)) e^{i(\omega_A - \omega_B)t} \right) \otimes f(t) \quad (27b)$$~~

Please replace paragraph [0166] with the following amended paragraph:

[0166] The accuracy of $E_{A\ est2}$ $E_{A\ est1}$, which uses crosstalk subtraction, can be compared with $E_{A\ est1}$ $E_{A\ est1}$, which does not utilize crosstalk subtraction. Substituting equations 26 into 27a provides:

$$E_{A\ est2}(t) = E_A(t) \otimes f(t) + \left(n(t) e^{-i\omega_A t} \right) \otimes f(t) + \left((E_B(t) - \underline{Q}(E_{B\ est1}(t))) e^{i(\omega_B - \omega_A)t} \right) \otimes f(t) \quad (28)$$

$$E_{A\ est2}(t) = E_A(t) \otimes f(t) + \left(n(t) e^{-i\omega_A t} \right) \otimes f(t) + \left((E_B(t) - \underline{Q}(E_{B\ est1}(t))) e^{i(\omega_B - \omega_A)t} \right) \otimes f(t) \quad (28)$$

Please replace paragraph [0167] with the following amended paragraph:

[0167] The function $\frac{E_B(t) - Q(E_{B\text{est}1}(t))}{E_B(t) - Q(E_{B\text{est}1}(t))}$ is zero most of the time, and has a pulse whenever a bit error occurs based on $E_{B\text{est}1}$. The noise term in equations 28 and 26a cannot be avoided, and in fact it is the target to make the system noise limited. Comparing equation 28 with 26a, $\frac{E_{A\text{est}2}}{E_{A\text{est}1}}$ is closer than $\frac{E_{A\text{est}1}}{E_{A\text{est}1}}$ to $\frac{E_A(t) \otimes f(t)}{E_A(t) \otimes f(t)}$ provided that $\frac{E_B - Q(E_{B\text{est}1})}{E_B - Q(E_{B\text{est}1})}$ has a lower root mean square (r.m.s.) deviation than $\frac{E_{B\text{est}1}}{E_{B\text{est}1}}$. This is correct when the BER of $E_{B\text{est}1}$ is lower than about 0.5. The channels should be spaced less than approximately the symbol rate before this condition is violated, so linear crosstalk subtraction enables a very low channel spacing to be achieved.

Please replace paragraph [0171] with the following amended paragraph:

[0171] The discussion above utilizes complex numbers to describe sine and cosine functions because this notation is a compact way of including the phase of the sine wave or cosine wave. For example the electric field is written in the form:

$$E(t) = \operatorname{Re}[E_s e^{i\omega t}] \quad (\text{A1})$$

$$E(t) = \operatorname{Re}[E_s e^{i\omega t}] \quad (\text{A1})$$

Please replace paragraph [0172] with the following amended paragraph:

[0172] where E_s is a complex number. This can be expressed in terms of sines and cosines as:

$$E(t) = \operatorname{Re}[E_s] \cos(\omega t) - \operatorname{Im}[E_s] \sin(\omega t)$$

$$E(t) = \operatorname{Re}[E_s] \cos(\omega t) - \operatorname{Im}[E_s] \sin(\omega t)$$

Please replace paragraph [0173] with the following amended paragraph:

[0173] Or if complex $\underline{E_s}$ E_s is written in terms of its magnitude and phase as:

$$\underline{E_s} = |E_s| e^{i\theta_s}$$

$$E_s = |E_s| e^{i\theta_s}$$

Please replace paragraph [0174] with the following amended paragraph:

[0174] then A1 becomes:

$$E(t) = |E_s| \cos(\omega t + \theta_s)$$

$$E(t) = |E_s| \cos(\omega t + \theta_s)$$

Please replace paragraph [0175] with the following amended paragraph:

[0175] The complex number notation is compact because the phase of the sine wave is stored in the phase of the complex number.

In some places in the discussion, there appear equations like:

$$\text{beat term} = \text{Re} \left[E_s E_{LO}^* e^{i\omega t} \right] \quad (\text{A2})$$

$$\text{beat term} = \text{Re} \left[E_s E_{LO}^* e^{i\omega t} \right] \quad (\text{A2})$$

Please replace paragraph [0176] with the following amended paragraph:

[0176] $\underline{E_{LO}}^*$ E_{LO}^* is the complex conjugate of $\underline{E_{LO}}$ E_{LO} , meaning that every occurrence of i or \underline{i} is replaced with $\underline{-i}$ or $-i$, and:

$$\underline{E_{LO}}^* = |E_{LO}| e^{-i\theta_{LO}}$$

$$E_{LO}^* = |E_{LO}| e^{-i\theta_{LO}}$$

Please replace paragraph [0177] with the following amended paragraph:

[0177] So A2 can be rewritten as:

$$\text{beat term} = |E_s| |E_{LO}| \cos(\omega t + \theta_s - \theta_{LO})$$

$$\text{beat term} = |E_s| |E_{LO}| \cos(\omega t + \theta_s - \theta_{LO})$$

Please replace paragraph [0178] with the following amended paragraph:

[0178] The appearance of $\underline{E_s} \underline{E_{LO}}^*$ in A2 means to take the phase difference between $\underline{E_s}$ and $\underline{E_{LO}}$.

Please replace paragraph [0179] with the following amended paragraph:

[0179] The power of an optical wave is given by the magnitude squared of the complex electric field, and does not have a sinusoid time dependence. In the case of a field given by A1:

$$\text{power} = (\underline{E_s e^{i\omega t}})^* (\underline{E_s e^{i\omega t}}) = |\underline{E_s}|^2$$

$$\text{power} = (\underline{E_s e^{i\omega t}})^* (\underline{E_s e^{i\omega t}}) = |\underline{E_s}|^2$$

Please replace paragraph [0183] with the following amended paragraph:

[0183] A Jones unit vector $\hat{\underline{p}}$ has the property that is:

$$\hat{\underline{p}} \cdot \hat{\underline{p}}^* = 1$$

$$\underline{\hat{\underline{p}} \cdot \hat{\underline{p}}^* = 1}$$

Please replace paragraph [0184] with the following amended paragraph:

[0184] If light polarized in SOP $\hat{\underline{p_T}} \hat{\underline{p_1}}$ passes through a polarizer oriented in direction $\hat{\underline{p_2}}$, then the electric field is scaled by $\hat{\underline{p_1}} \cdot \hat{\underline{p_2}}^* \hat{\underline{p_1}} \cdot \hat{\underline{p_2}}^*$. In general $0 \leq |\hat{\underline{p_T}} \cdot \hat{\underline{p_2}}^*| \leq 1$ $0 \leq |\hat{\underline{p_1}} \cdot \hat{\underline{p_2}}^*| \leq 1$.

Please replace paragraph [0185] with the following amended paragraph:

[0185] The Jones unit vector of the state orthogonal to $\hat{\mathbf{p}} \underline{\hat{\mathbf{p}}}$ is denoted in the above discussion by $\hat{\mathbf{p}}^\dagger \underline{\hat{\mathbf{p}}^\dagger}$, and

$$\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}^{\dagger *} = 0$$

$$\underline{\hat{\mathbf{p}}} \cdot \hat{\mathbf{p}}^{\dagger *} = 0$$

Please replace paragraph [0186] with the following amended paragraph:

[0186] If

$$\hat{\mathbf{p}} = \begin{pmatrix} \hat{p}_x \\ \hat{p}_y \end{pmatrix}$$

$$\underline{\hat{\mathbf{p}}} = \begin{pmatrix} \hat{p}_x \\ \hat{p}_y \end{pmatrix}$$

then

$$\hat{\mathbf{p}}^\dagger = \begin{pmatrix} \left| \frac{\hat{p}_y}{\hat{p}_x} \right| \hat{p}_x \\ -\left| \frac{\hat{p}_x}{\hat{p}_y} \right| \hat{p}_y \end{pmatrix}$$

$$\underline{\hat{\mathbf{p}}}^\dagger = \begin{pmatrix} \left| \frac{\hat{p}_y}{\hat{p}_x} \right| \hat{p}_x \\ -\left| \frac{\hat{p}_x}{\hat{p}_y} \right| \hat{p}_y \end{pmatrix}$$